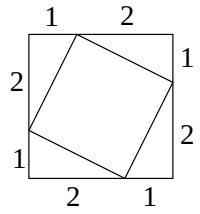
Sample exam 2
Math 10, Spring 2004

1. Find the area of the inner square in the picture below. Show all your work.

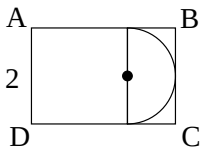


2. Is it possible that

$$0 - 70530 - 59056 - 0$$

is a valid Universal Product Code? Justify your answer, and show all your work.

3. Below we have a picture of a square of side length 2, a half-circle whose center is at the midpoint of one of the sides of that square, and a rectangle $ABCD$ that extends the square and is just large enough to contain the half-circle.



What is the ratio between the lengths of the long and short sides of $ABCD$? Is $ABCD$ a Golden Rectangle? Briefly justify your answer and show all your work.

4. Let OEO be the set of all odd numbers except one, i.e., let $OEO = \{3, 5, 7, 9, \dots\}$. Does OEO have the same cardinality as the natural numbers? If yes, describe a one-to-one correspondence that justifies your answer; if no, explain why not.

5. For each of the following numbers, say whether the number is rational or irrational, and briefly justify your answer. If a proof would be required to justify your answer completely, you don't need to explain that proof; just state that a proof is required.

   (a) $\dfrac{244\sqrt{7}}{20\sqrt{7}}$.

   (b) $\sqrt{13}$.

   (c) $13.1452013013013\dots$, where the 013013 pattern continues.

6. In each part of this question, give a brief explanation in a complete sentence or sentences.

   (a) In **ONE** sentence, explain precisely what Cantor's Theorem says.

(b) To prove Cantor's Theorem, we have to explain why a certain thing doesn't exist. What is that thing, and how does its nonexistence imply Cantor's Theorem? (Your answer should take no more than 3–5 sentences.)

(c) The details of Cantor's Theorem involve constructing a certain real number M whose digits after the decimal point are (for example) either 1 or 7. Briefly explain how M can be constructed, using the following list of real numbers to begin an example of this construction: $2.71828\ldots$, 1.21567, $12.333333\ldots$.